Section One: Calculator-free 35% (53 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 (6 marks)

The polynomial $h(z) = z^4 - 4z^3 + 17z^2 - 16z + a$, where a is a real constant, is exactly divisible by $(z - 2i)$.

(a) Determine the value of a. (2 marks)

(b) Write down two zeros (roots) of $h(z)$. (1 mark)

(c) Determine the other zeros of $h(z)$. (3 marks)

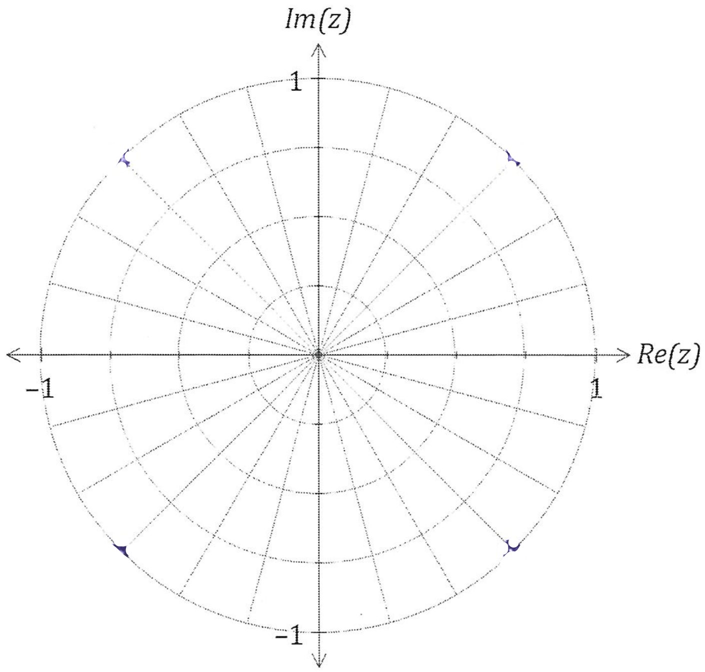
Question 2 **(5 marks)**

Let $v = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.

(a) Express v in polar form. (2 marks)

(b) Show that $v^4 = -1$. (1 mark)

(c) Plot the roots of $z^4 + 1 = 0$ on the following Argand diagram. (2 marks)

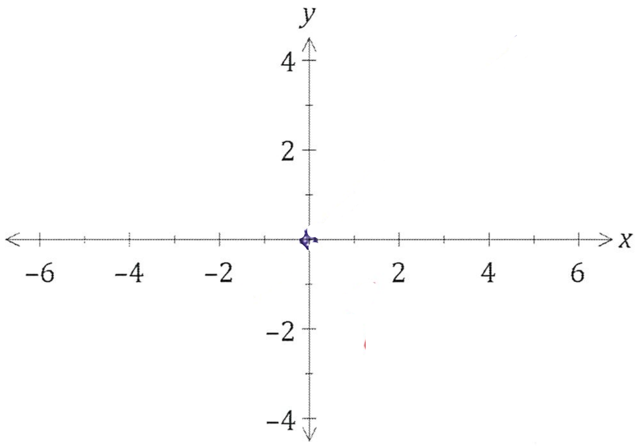


Question 3 **(8 marks)**

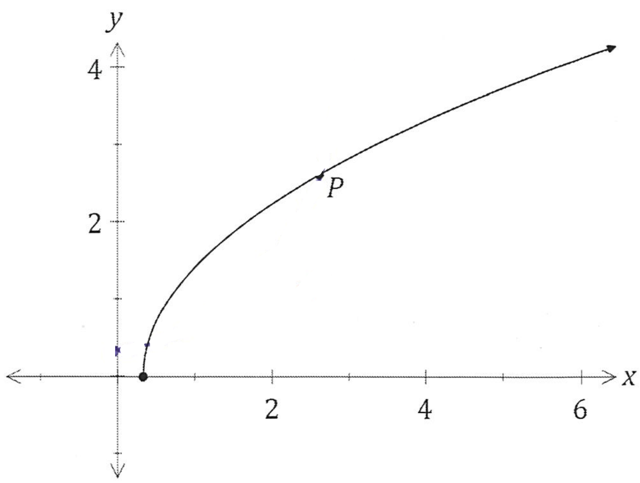
Two functions are defined by $f(x) = \sqrt{3x - 1}$ and $g(x) = \frac{1}{x}$.

(a) Determine the composite function $f(g(x))$ and the domain over which it is defined. (3 marks)

(b) Sketch the graph of $y = g(g(x))$ on the axes below. (2 marks)



(d) The graph of $y = f(x) = \sqrt{3x - 1}$ is shown below, passing through point P with coordinates $(2.62, 2.62)$.
Determine an equation for $f^{-1}(x)$, the inverse of $f(x)$, and sketch the graph of $y = f^{-1}(x)$ on the same axes. (3 marks)

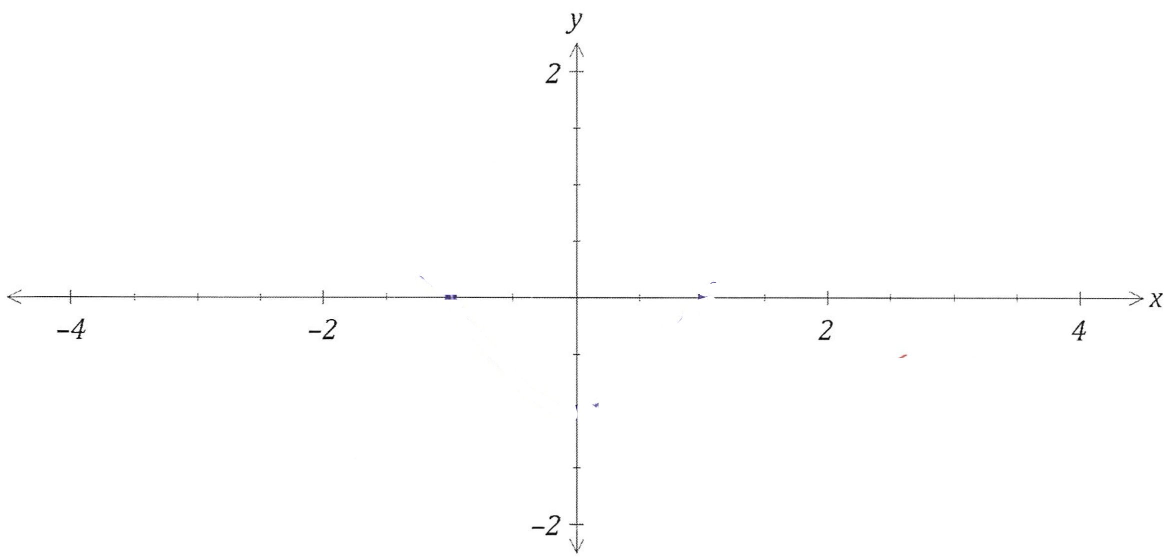


Question 4 **(8 marks)**

The function f is defined as $f(x) = \frac{x^2-1}{x^2+1}$.

(a) Show that the **only** stationary point of the function occurs when $x = 0$. (2 marks)

(b) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)



(c) Using your graph, or otherwise, determine all solutions to

(i) $f(x) = |f(x)|$. (1 mark)

(ii) $f(x) = f(|x|)$. (1 mark)

(iii) $f(x) = \frac{1}{f(x)}$. (1 mark)

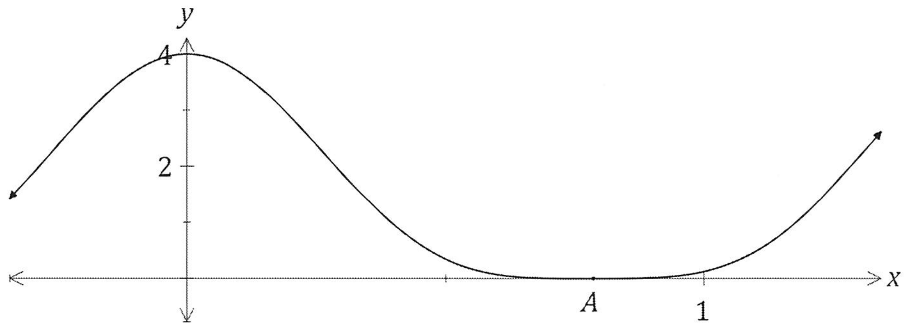
Question 5 **(7 marks)**

(a) Using partial fractions, or otherwise, determine $\int \frac{x-19}{(x+1)(x-4)}\,dx$. (4 marks)

(b) Use the substitution $u = \sin x$ to evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}}\,dx$. (3 marks)

Question 6 **(7 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = 4\cos^4(2x)$ and A is the smallest root of $f(x),\ x > 0$.



(a) Show that $4\cos^4(2x) = \frac{3+4\cos(4x)+\cos(8x)}{2}$. (3 marks)

(b) Hence determine $\int 4\cos^4(2x)\, dx$. (2 marks)

(c) Use the formula $V_x = \pi\int_a^b y^2 dx$ to write a definite integral to represent the volume of the solid generated when the region bounded by $= f(x)$, $y = 0$, $x = 0$ and $x = A$ is rotated through 360° about the x-axis. (2 marks)

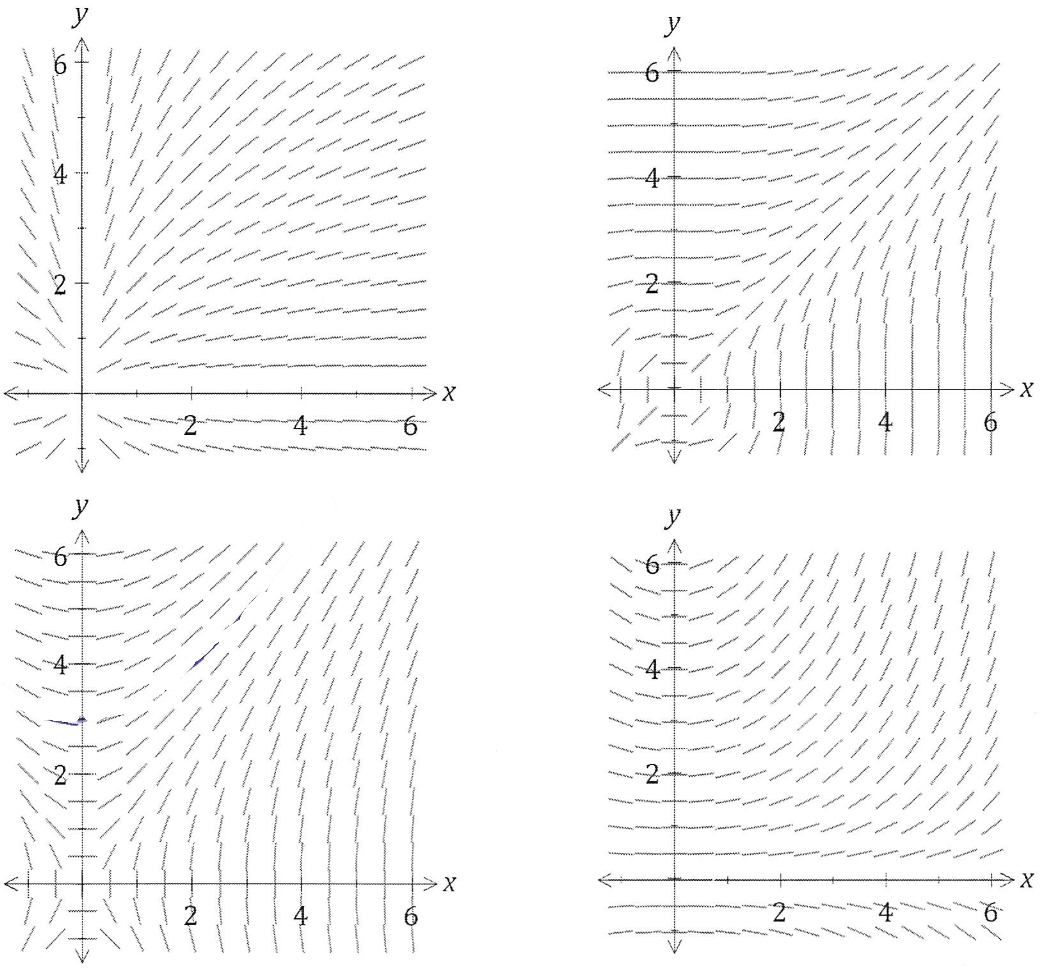
Question 7 **(7 marks)**

(a) Show that the gradient of the curve $2x^2 + y^2 = 3xy$ at the point $(1, 2)$ is 2. (3 marks)

(b) Another curve passing through the point $(-2, 10)$ has gradient given by $\frac{dy}{dx} = \frac{2xy}{1+x^2}$.
Use a method involving separation of variables and integration to determine the equation of the curve. (4 marks)

Question 8 **(5 marks)**

The differential equation $y' = \frac{2x}{y}$ is shown in just one of the four slope fields below.



(a) On the slope field for $y' = \frac{2x}{y}$, sketch the solution of the differential equation that passes through the point $(2, 4)$. (3 marks)

(b) Another solution to the differential equation passes through the point $(6, -3)$. Use the incremental formula $\delta y \approx \frac{dy}{dx} \times \delta x$, with $\delta x = \frac{1}{10}$, to estimate the y-coordinate of this curve when $x = 6.1$. (2 marks)

End of questions